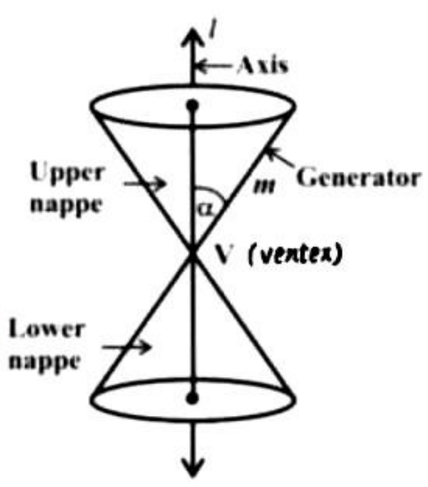
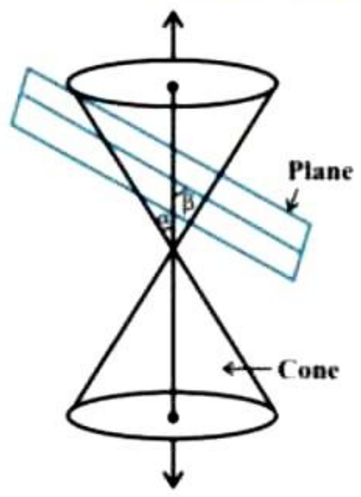
Conic Sections

Sections of a cone :

The intersection of a plane with a cone, the section so obtained is called a conic section.





Circle, ellipse, parabola and hyperbola :

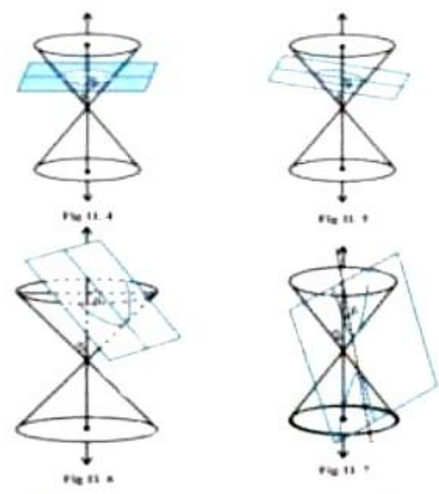
When the plane cuts nappe of the cone, we have the following situations :

(a) When $\beta = 90^\circ$, the section is a circle.

(b) When $\alpha < \beta < 90^\circ$, the section is an ellipse.

(c) When $\beta = \alpha$; the section is a parabola.

(d) When $0 \le \beta < \alpha$; the plane cuts through both the nappes and the curves of intersection is a hyperbola.



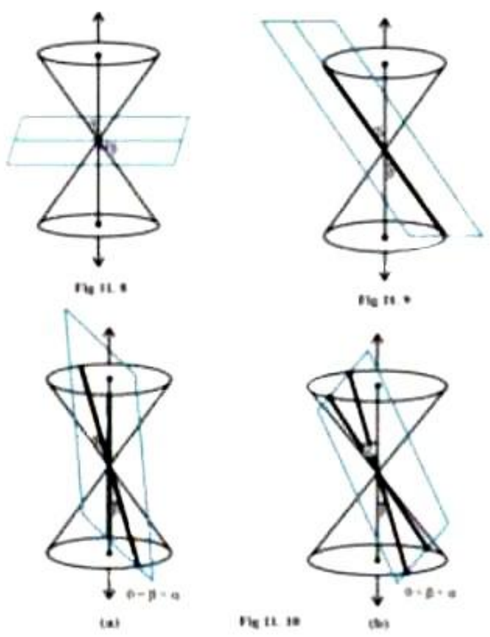
Degenerated conic sections :

When the plane cuts at the vertex of the cone, we have the following different cases :

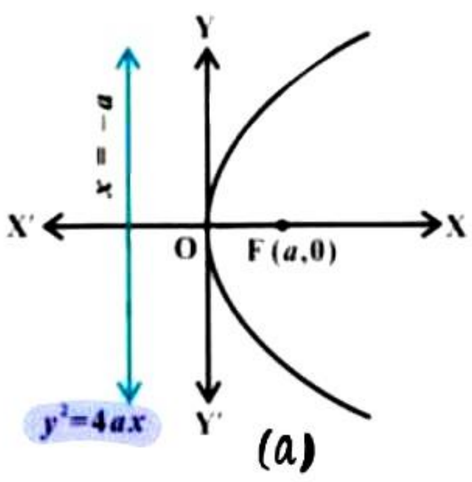
(a) When $\alpha < \beta \le 90^\circ$, then the section is a point.

(b) When $\beta = \alpha$, the plane contains a generator of the cone and the section is a straight line. It is the degenerated case of a parabola.

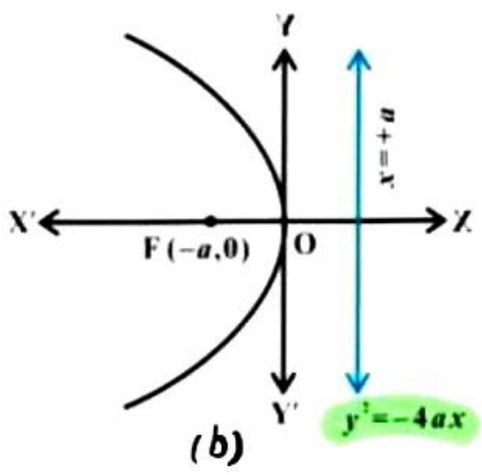
(c) When $0 \le \beta < \alpha$, the section is a pair of intersecting straight lines. It is degenerated case of a hyperbola.



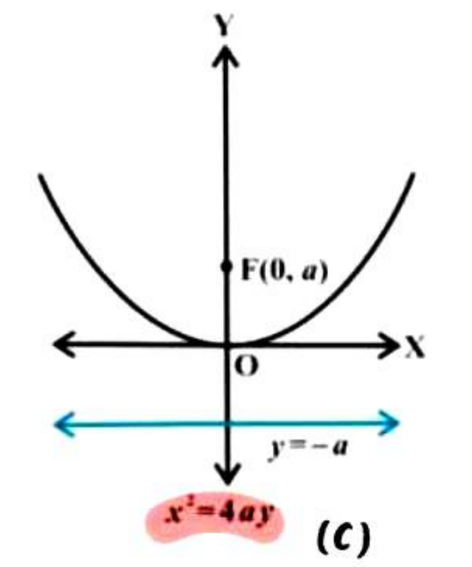
Standard equation of parabola



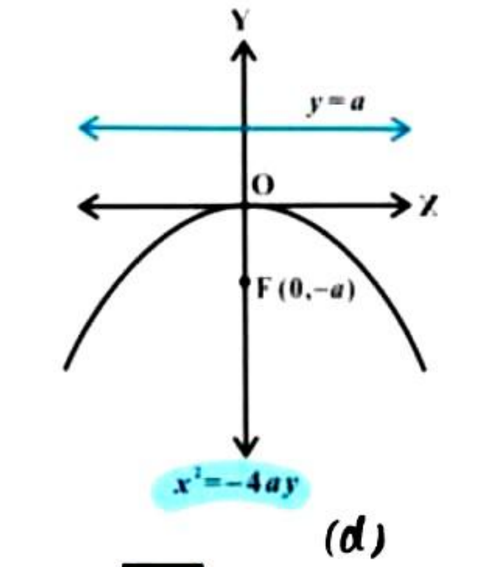
(a)



(b)



(c)



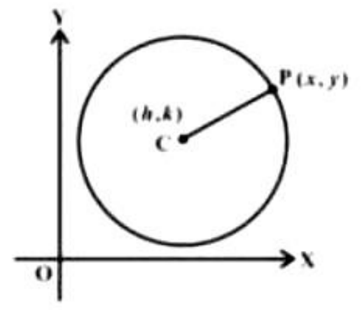
(d)

Circle equation

$$(x-h)^2 + (y-k)^2 = r^2$$

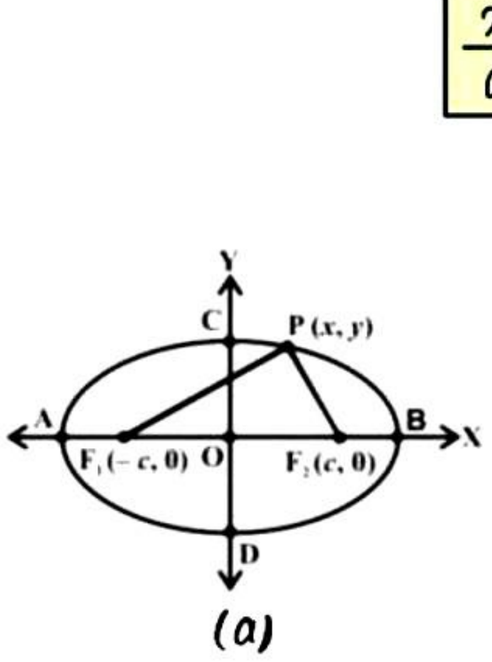
centre at (h, k)

circle radius = r

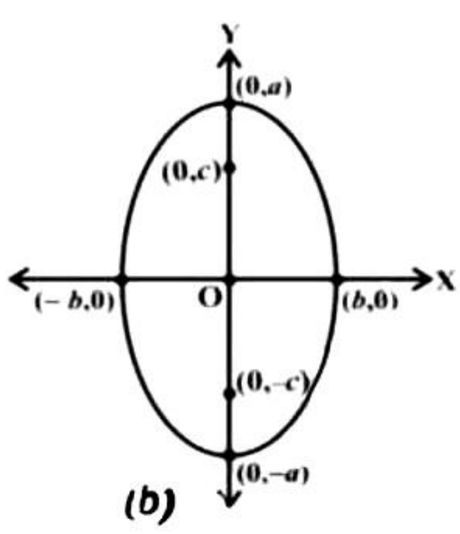


Standard equations of an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



(a)



(b)

Latus rectum of parabola

$4a$

Latus rectum of ellipse

$\frac{2b^2}{a}$

The eccentricity of an ellipse

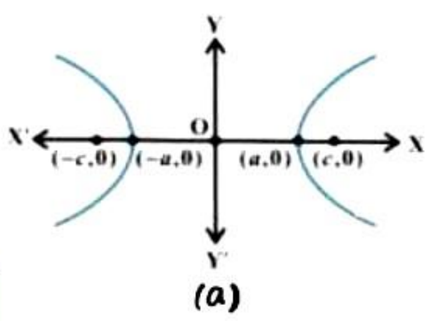
$e = \frac{c}{a}$ — distance from the centre

- Relationship between semi-major axis, semi minor axis and the distance of the focus from the centre of the ellipse.

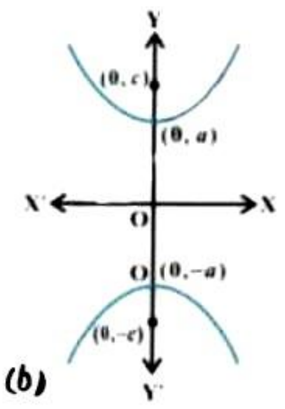
$$a^2 = b^2 + c^2 \quad \text{OR} \quad c = \sqrt{a^2 - b^2}$$

- Standard equation of hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



(a)



(b)

- Latus rectum of hyperbola $\frac{2b^2}{a}$

- The eccentricity of an hyperbola $e = \frac{c}{a}$ → distance from the centre

- **Note** : A hyperbola in which $a = b$ is called an equilateral hyperbola.